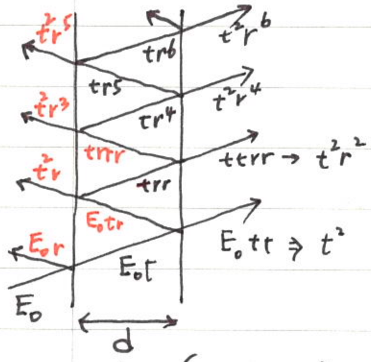
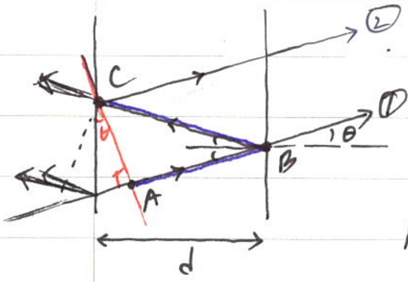
CH.4 Multiple-beam Interference



$$E_0 = E_0\left[(t^2 + t^2r^2 + t^2r^4 + \cdots) + (r + t^2r + t^2r^3 + t^2r^5 + \cdots)\right]$$

$$\equiv E'_{0T} + E'_{0R}$$

Concerning the phase shift of δ for each interval,

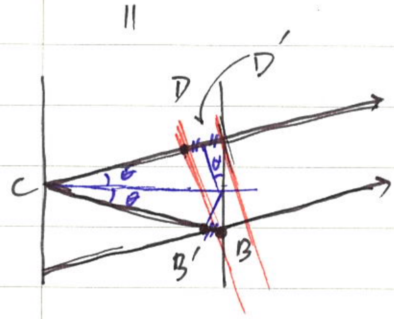


The path difference Δ btwn ① & ②:

$$\Delta = \overline{AB} + \overline{BC}$$

$$\left(\overline{AB} + \overline{BC}\right) = \left(\overline{BC} + \overline{CD}\right)$$

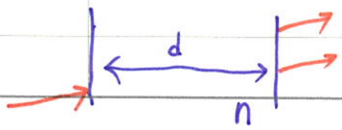
||



$$= \overline{B'C} + \overline{CD'}$$

$$= \underline{2d\cos\theta} = \Delta$$

HW#5. Prove $\Delta = 2d\cos\theta$ for $n = 1.5$ of the slab in the air.



∴ The phase shift δ each interval is

$$\delta = 2kd\cos\theta = \frac{4\pi}{\lambda} d\cos\theta$$

(i) By rewritting E_T' with δ:

$$E_T = E_0 t^2 \left(\underbrace{1 + r^2 e^{i\delta} + r^4 e^{2i\delta} + \cdots}_{1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}} \right)$$

$$\therefore E_T = \frac{E_0 t^2}{1 - r^2 e^{i\delta}}$$

The transmitted intensity I_T:

$$I_T = |E_T|^2 = I_0 \left(\frac{t^4}{(1 - r^2 e^{i\delta})(1 - e^{-i\delta} r^2)} \right)$$

$$= I_0 \frac{(1-r^2)^2}{1 + r^4 - 2r^2\cos\delta}$$

(ii) By considering phase shift δ_r on the reflection,

$$r = |r| e^{i\delta_r/2}$$: amplitude vs. intensity

$$\rightarrow \quad R = |r|^2 = rr^*$$

$$I_T = (E_T E_T^*) = I_0 \frac{T^2}{|1 - Re^{i\Delta}|^2},$$

$$\Delta = \delta + \delta_r$$

$$|1-Re^{i\Delta}|^2 = (1-Re^{i\Delta})(1-Re^{-i\Delta})$$
$$= 1 - R(e^{i\Delta}+e^{-i\Delta}) + R^2$$
$$= 1 - 2R\cos\Delta + R^2$$

$$\cos\Delta = \cos\left(\frac{\Delta}{2}+\frac{\Delta}{2}\right) = \cos^2\frac{\Delta}{2} - \sin^2\frac{\Delta}{2} = 1 - 2\sin^2\frac{\Delta}{2}$$

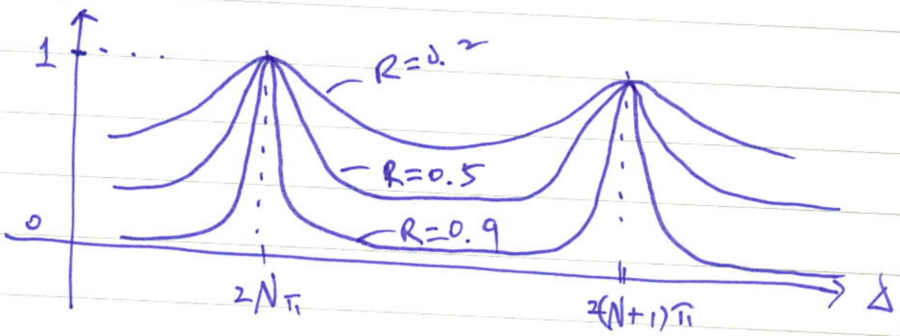
$$\therefore\ |1-Re^{i\Delta}|^2 = 1 - 2R\left(1-2\sin^2\frac{\Delta}{2}\right) + R^2$$
$$= (1-2R+R^2) + 4R\sin^2\frac{\Delta}{2}$$
$$= (1-R)^2\left(1+\frac{4R}{(1-R)^2}\sin^2\frac{\Delta}{2}\right)$$

$$\therefore\ I_T = I_0\frac{|t|^4}{|1-r^2e^{i\delta}|^2} = I_0\frac{T^2}{|1-Re^{i\Delta}|^2}$$

$$= I_0\frac{T^2}{(1-R^2)^2}\,\frac{1}{\left(1+F\sin^2\frac{\Delta}{2}\right)}$$

where $\dfrac{1}{1+F\sin^2(\frac{\Delta}{2})}$ is Airy fn. Δ

$F = \dfrac{4R}{(1-R)^2}$. is coefficient of finesse.



→ Airy fn goes to max if $\frac{\Delta}{2} = N\pi$ regardless of F.

→ $F\uparrow$, the fringe pattern sharpens!

For maximum I_T,

$$2N\pi = \Delta = \delta + \delta_r$$

$$= 2kd\cos\theta + \delta_r = \frac{4\pi}{\lambda} nd\cos\theta + \delta_r$$

- For maximum transmission,

$$\Delta = 2m\pi \quad \& \quad I_T = I_0 \frac{T^2}{(1-R)^2}$$

- For minimum transmission,

$$\Delta = m\pi \quad \& \quad I_T = I_0 \frac{T^2}{(1-R)^2} \frac{1}{1+F}$$

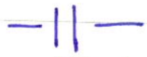
$$\left(\frac{1}{1+F} = \frac{1}{1 + \frac{4R}{(1-R)^2}} = \frac{(1-R)^2}{(1+R)^2} \right)$$

$$\therefore \quad I_T = I_0 \frac{T^2}{(1+R)^2}$$

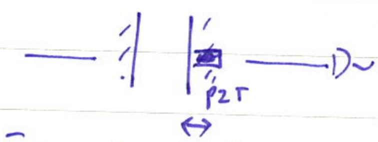
- For lossless medium,

$$R + T = 1.$$

In 1899, Fabry & Perot employed multiple beam interference. to measure a wavelength.

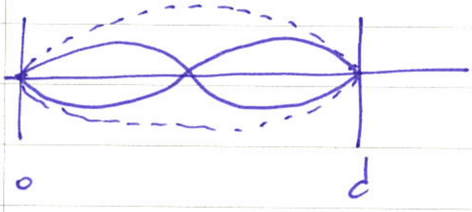
- F-P etalon 
(fixed)

- Scanning F-P interferometer



- Free spectral range $\delta_{fsr} \Rightarrow \Delta_{N+1} - \Delta_N = 2\pi$

$$\nu_{N+1} - \nu_N = \frac{c}{2nd}$$



$$\frac{\lambda}{2} m = d$$

$$\cancel{(m+1) - (m) = \frac{2d}{\lambda}}$$

$$f\lambda = c/n \rightarrow \frac{1}{\lambda} = \frac{nf}{c}$$

$$\cancel{(m+1) - (m) =}$$

$$\frac{\lambda_m}{2} \cdot m = d \quad ; \quad \frac{\lambda_{m+1}}{2}(m+1) = d$$

$$m = \frac{2d}{\lambda_m} \quad \text{or} \quad \lambda_m = \boxed{\frac{2d}{m} = \frac{c}{nf_m}}$$

$$m+1 = \frac{2d}{\lambda_{m+1}} \quad \text{or} \quad \lambda_{m+1} = \boxed{\frac{2d}{m+1} = \frac{c}{nf_{m+1}}}$$

$$f_{m+1} = \frac{c}{2nd}(m+1)$$

$$f_m = \frac{c}{2nd}(m)$$

$$\therefore \quad f_{m+1} - f_m = \frac{c}{2nd} \qquad (4.16)$$

$\rightarrow$ $d \uparrow$, $\Delta f \downarrow$ (resolution $\uparrow$)

ex) $d = 3.0$ cm ; $n = 1$; $c = 3 \times 10^8$ m

$$\Delta f = \frac{3 \cdot 10^8}{(0.03)(2)} = \underline{5 \text{ GHz}}$$

$d = 1$ m

$$\Delta f = \frac{3 \cdot 10^8}{2} = \underline{150 \text{ MHz}}$$

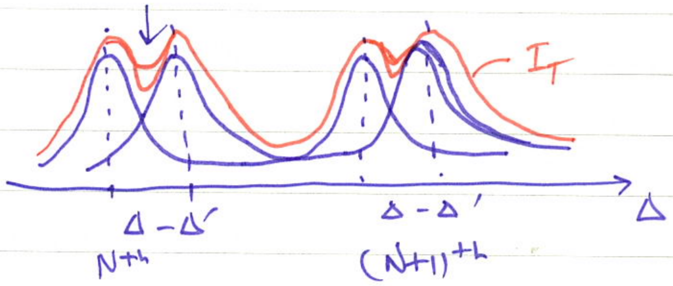
4.3 Resolution of F-P instruments

ω & ω', $|\omega' - \omega| \ll \omega$

$$I_T = I_0\left[\left(1+F\sin^2\frac{\Delta}{2}\right)^{-1} + \left(1+F\sin^2\frac{\Delta'}{2}\right)^{-1}\right],$$

$$\left(\begin{array}{l} \Delta = \delta_r + 2kd = \delta_r + \frac{2\omega d}{c} \\ \Delta' = \delta_r + 2k'd = \delta_r + \frac{2\omega' d}{c} \end{array}\right. \quad ; \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

· Resolution criteria : Talyor criterion
→ each intensity curve crosses the half intensity point.



→ At the saddle point (red), total superposed intensity I_T must be I_0

→ The saddle point is the midpoint of $\Delta - \Delta'$.

→ $\frac{\Delta - \Delta'}{2}$

∴ The superposed intensity can be written by

$$F\sin^2\left(\frac{\Delta-\Delta'}{4}\right) \rightarrow I = 2I_0\left(1+F\sin^2\left(\frac{\Delta-\Delta'}{4}\right)\right)^{-1}$$

For very small $(\Delta - \Delta')$,

$$F\sin^2\left(\frac{\Delta-\Delta'}{4}\right) \; \sqrt{F}\frac{|\Delta-\Delta'|}{4} = 1 \quad \therefore |\Delta - \Delta'| = 4/\sqrt{F}$$

· $\sin^2 x \to x^2$

From (4-9), $F = \frac{4R}{(1-R)^2}$ (coefficient of Finesse)

$\sqrt{F} = \frac{2\sqrt{R}}{1-R}$: Finesse $\mathcal{F}$ ($\Rightarrow \frac{\pi\sqrt{F}}{2} = \frac{\pi\sqrt{R}}{1-R}$)

$\therefore \quad |\Delta - \Delta'| = \frac{2(1-R)}{\sqrt{R}}$ (4.18)

From (4.7), $\Delta = \delta + \delta_r \doteq 2k n'' d = \frac{2\omega d}{c}$

$|\omega - \omega'| = \frac{2c}{d}\sqrt{F}$ $\left(4/\sqrt{F}\right)_{\Delta - \Delta'}$

$= \frac{c}{d}\left(\frac{1-R}{\sqrt{R}}\right)$

· Ratio of the free spectral range to the fringe width :

$\mathcal{F} = \frac{\Delta_{N+1} - \Delta_N}{|\Delta - \Delta'|} = \frac{2\pi}{\frac{2(1-R)}{\sqrt{R}}} = \pi\left(\frac{\sqrt{R}}{1-R}\right)$

· Resolving power, RP :

$RP = \frac{\omega}{\delta\omega} = \frac{\nu}{\delta\nu} = \frac{\lambda}{d\lambda} = N\mathcal{F}$

($\mathcal{F}$: $\frac{\text{separation btwn transmission peaks}}{\text{FWHM of the peak}}$)