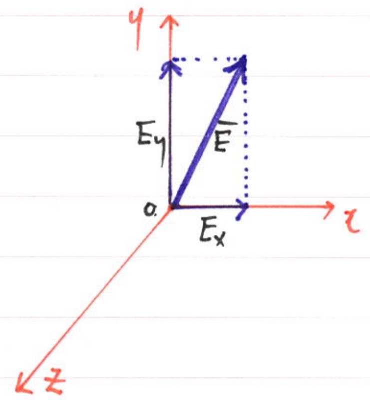
PAGE

DATE 6/13/2016

CH. 14 Matrix treatment of Polarization

14-1 Mathematical representation of polarized light : Jones vectors



· $\bar{E} = E_x \hat{x} + E_y \hat{y}$

In complex notation,

$$\begin{cases} \tilde{E}_x = E_{ox} e^{i(kz - \omega t + \phi_x)}, \\ \tilde{E}_y = E_{oy} e^{i(kz - \omega t + \phi_y)}, \end{cases}$$

where $E_x = Re(\tilde{E}_x)$; $E_y = Re(\tilde{E}_y)$

$$\rightarrow \quad \tilde{E} = E_{ox} e^{i(kz-\omega t+\phi_x)} \hat{x} + E_{oy} e^{i(kz-\omega t+\phi_y)} \hat{y}$$

$$= [E_{ox} e^{i\phi_x} \hat{x} + E_{oy} e^{i\phi_y} \hat{y}] e^{i(kz-\omega t)} = \tilde{E}_0 e^{i(kz-\omega t)}$$

complex amplitude

· Jones vector : 2-D matrix notation

$$\tilde{E}_0 = \begin{bmatrix} \tilde{E}_{ox} \\ \tilde{E}_{oy} \end{bmatrix} = \begin{bmatrix} E_{ox} e^{i\phi_x} \\ E_{oy} e^{i\phi_y} \end{bmatrix}$$

(i) Linear polarization along $\hat{y}$

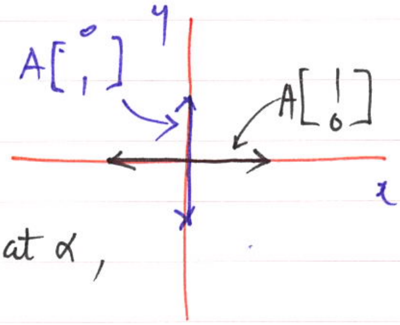
$$\tilde{E}_0 = \begin{bmatrix} E_{ox} e^{i\phi_x} \\ E_{oy} e^{i\phi_y} \end{bmatrix} = \begin{bmatrix} 0 \\ A \end{bmatrix} = A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

for $E_{ox} = 0$, $E_{oy} = A$, $\phi_y = 0$

Here, in $\begin{bmatrix} a \\ b \end{bmatrix}$, $a^2 + b^2 = 1$ for normalization.

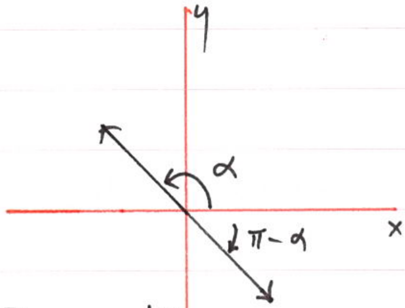
ex) Linear polarization along $\hat{x}$

$$\tilde{E}_0 = A\begin{bmatrix}1\\0\end{bmatrix}$$



ex) Linear polarization in general at α,

$$\tilde{E}_0 = A\begin{bmatrix}\cos\alpha\\ \sin\alpha\end{bmatrix}$$



For $\alpha = 60°$

$$\begin{bmatrix}\cos 60\\ \sin 60\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\ \sqrt{3}\end{bmatrix}$$

$$\rightarrow \alpha = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{E_{0y}}{E_{0x}}\right)$$

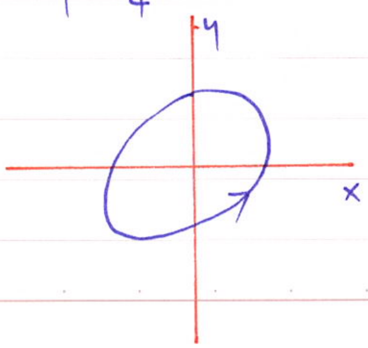
ex) elliptical polarization

$\rightarrow \Delta\phi = \phi_y - \phi_x \neq 0, \pi$ & $E_{0x} \neq E_{0y}$

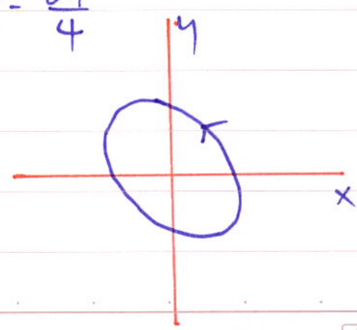
See Lissjous figure in Fig. 14-4

(i) $\Delta\phi = \frac{\pi}{4}$

(ii) $\Delta\phi = \frac{3\pi}{4}$



$\begin{bmatrix}\cos\alpha\\ \sin\alpha\end{bmatrix}$

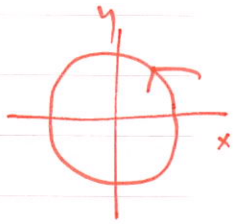


· For $\begin{cases} E_{ox} = E_{oy} = A \\ E_x \text{ leads } E_y \text{ by } \frac{\pi}{2}. \end{cases}$ $\rightarrow \phi_y > \phi_x$

$\Rightarrow$ Let $\phi_x = 0$, $\phi_y = \epsilon \left(\frac{\pi}{2}\right)$

then $\begin{cases} \tilde{E}_x = E_{ox} e^{-i\omega t} \\ \tilde{E}_y = E_{oy} e^{-i(\omega t - \epsilon)} \end{cases}$ $\rightarrow$ lag ϵ in $\hat{y}$, $\epsilon = \frac{\pi}{2}$

or $\begin{cases} E_x = A\cos\omega t \\ E_y = A\cos(\omega t - \frac{\pi}{2}) = A\sin\omega t \end{cases}$



In Jones vector notation,

$$\tilde{E}_0 = \begin{bmatrix} E_{ox} e^{i\phi_x} \\ E_{oy} e^{i\phi_y} \end{bmatrix} = \begin{bmatrix} A \\ A e^{i\frac{\pi}{2}} \end{bmatrix} = A \begin{bmatrix} 1 \\ i \end{bmatrix}.$$

counterclockwise rotation!

For normalization,

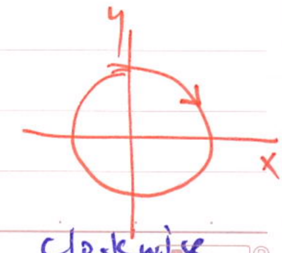
$1^2 + |i|^2 = 2$. $\therefore A = \frac{1}{\sqrt{2}}$

$\rightarrow \tilde{E}_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$, Left Circularly Polarized (LCP)

Likewise,

$$\tilde{E}_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}, \text{ RCP}$$

$\rightarrow E_x$ lags E_y by $\frac{\pi}{2}$



Clockwise rotation!

ex) $\begin{bmatrix} 2i \\ 2 \end{bmatrix}$, What is the polarization ?

$$\rightarrow \begin{bmatrix} 2i \\ 2 \end{bmatrix} = 2\begin{bmatrix} i \\ 1 \end{bmatrix} = \underline{2i}\begin{bmatrix} 1 \\ -i \end{bmatrix} \rightarrow \therefore \text{RCP}$$

prefactor (Ignore prefactor for polarization determination.)

ex) $\begin{bmatrix} A \\ B+iC \end{bmatrix}$, What is the polarization ?

$$\rightarrow \tilde{E}_0 = \begin{bmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{bmatrix} = \begin{bmatrix} A \\ be^{i\epsilon} \end{bmatrix}, \quad \phi_y - \phi_x = \epsilon, \ \phi_x = 0$$

Here, $be^{i\epsilon} = b(\cos\epsilon + i\sin\epsilon) = B + iC$

$\rightarrow$ E_x leads E_y by ϵ. $\rightarrow$ counterclockwise rotation!

☆ Usefulness of Jones vector : Superposition of polarized modes

ex) $\begin{bmatrix} 1 \\ i \end{bmatrix} + \begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$: Linear in $\hat{x}$

LCP ↺ RCP ↻ $\leftrightarrow$

ex) $\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$: 45° Linear polarization

$\hat{y}$ $\hat{x}$ $\alpha = 45^\circ$

$\updownarrow$ $\leftrightarrow$ $\nearrow$

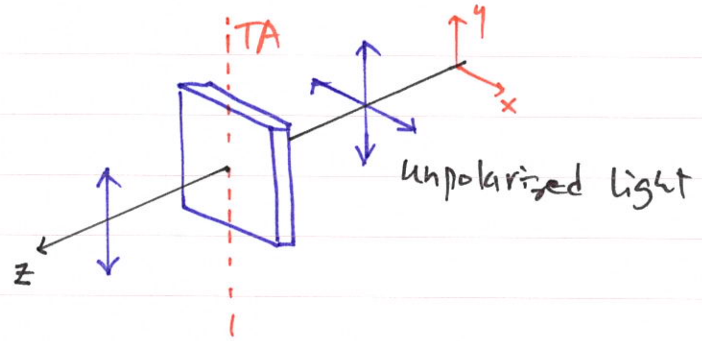
14.2 Jones Matrix

- To represent polarizers, how to modify light polarization.

$$\bar{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(i) Linear Polarizer : to transmit in a polarization direction

TA : transmission axis

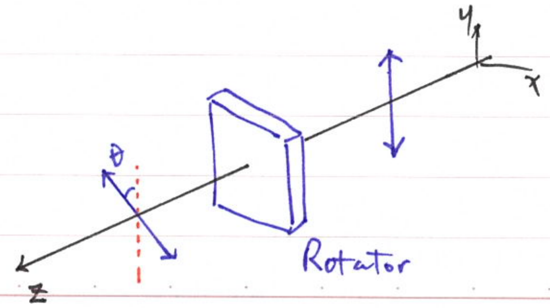


(ii) Phase Retarder

- To introduce phase difference $\Delta\phi$ by retardation (slowing speed)
- For $\Delta\phi = \frac{\pi}{2}$, quarter-wave plate.
  $\Delta\phi = \pi$, half-wave plate

(iii) Rotator

- To rotate the direction of linearly polarized light.



PAGE
DATE

ex)

(i) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, vertical → vertical pass !

Input   Output

(ii) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, horizontal → block !

Q. Find out linear polarizer Jones matrix M.

Sol) from (i), $a(0)+b(1)=0$; $c(0)+d(1)=1$

$\rightarrow b=0$; $d=1$

From (ii), $a(1)+b(0)=0$; $c(1)+d(0)=0$

$\rightarrow a=0$; $c=0$

$$\therefore M = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$, TA vertical linear polarizer

· TA at θ linear polarizer M,

$$M = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

· phase retarder M,

$$M = \begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix}$$, ϵ_x & ϵ_y : phase advance

· QWP, SA vertical ($\epsilon_y - \epsilon_x = \frac{\pi}{2}$)

$$M = \begin{bmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} = e^{-i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

· Rotator at β

$$M = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

< See Table 14-2. >