Group delay τ :

$$\tau = \frac{L}{v_g} = L\frac{dk}{d\omega} , \qquad k = \frac{2\pi n}{\lambda} ; \omega = 2\pi f$$

(i) From $\omega = \frac{2\pi c}{\lambda}$

$$d\omega = -\frac{2\pi c}{\lambda^2} d\lambda$$

$$\rightarrow \frac{1}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{1}{d\lambda}$$

$$\therefore \tau = -\frac{L\lambda^2}{2\pi c}\frac{dk}{d\lambda} , \qquad \frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}n + \frac{2\pi}{\lambda}\frac{dn}{d\lambda}$$

$$= -\frac{2\pi}{\lambda^2}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

$$= \frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

(ii) From $k = \frac{\omega}{c} \cdot n$,

$$\frac{dk}{d\omega} = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}$$

$$= \frac{n}{c} + \frac{\omega}{c}\left(-\frac{\lambda^2}{2\pi c}\right)\frac{dn}{d\lambda}$$

$$= \frac{1}{c}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

$$\therefore \tau = \frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

morning glory

6.5 Propagation of light in conducting media

- Free electrons in a metal
  - $\rightarrow$ no bound electrons
  - $\rightarrow$ no elastic restoring force due to $\bar{E}$

- $\bar{F} = -e\bar{E} = m\dot{\bar{v}} + m\tau^{-1}\underline{\bar{v}}$ ← $\bar{e}$ velocity

  $m\tau^{-1}$ ↳ frictional dissipation const.

- Current density by $\bar{v}$,

$$\bar{J} = -Ne\bar{v} \quad , \quad N : \#\text{ of } \bar{e}/\text{vol.}$$

$$\rightarrow \bar{v} = -\frac{1}{Ne}\bar{J}$$

$$\therefore \bar{F} = -e\bar{E} = -m\left(\frac{1}{Ne}\right)\dot{\bar{J}} + m\tau^{-1}\left(-\frac{1}{Ne}\right)\bar{J}$$

$$\rightarrow \boxed{\frac{Ne^2}{m}\bar{E} = \dot{\bar{J}} + \tau^{-1}\bar{J}}$$

- Decay of transient current :

From $\dot{\bar{J}} + \tau^{-1}\bar{J} = 0$,

$$\frac{\dot{J}}{J} = -\tau \rightarrow \int \frac{\dot{J}}{J}\,dt = -\int \tau^{-1} dt$$

$\nearrow \ln J \qquad \nearrow -\left.\frac{t}{\tau}\right|_0^t + c$

$$\rightarrow J(t) = J_0\, e^{-t/\tau}$$

τ : relaxation time

- For a static electric field,

$\dot{\bar{J}} = 0$ due to $\bar{J} = \sigma\bar{E}$.

$$\therefore \tau^{-1}\bar{J} = \frac{Ne^2}{m}\bar{E} \qquad \therefore \sigma = \frac{Ne^2}{m}\tau$$

static conductivity σ.

· For harmonic oscillating $\bar{E} = E_0 e^{-i\omega t}$,

· $\bar{J} = \sigma \bar{E}$

$$\dot{\bar{J}} = \sigma \dot{\bar{E}} = -i\omega\sigma E$$

i) From $\dot{\bar{J}} + \tau^{-1}\bar{J} = \frac{Ne^2}{m}\bar{E}$, & $\boxed{\sigma = \frac{Ne^2}{m}\tau}$

$$(-i\omega + \tau^{-1})\bar{J} = \frac{Ne^2}{m}\bar{E} = \tau^{-1}\sigma\bar{E}$$

$$\therefore \bar{J} = \left(\frac{\tau^{-1}}{\tau^{-1} - i\omega}\right)\sigma\bar{E} = \left(\frac{\sigma}{1 - i\omega\tau}\right)\bar{E}$$

ii) From (6.14) $\underbrace{\nabla \times (\nabla \times \bar{E})}_{-\nabla^2 \bar{E}} + \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2} = \cancel{-\mu_0 \frac{\partial^2 \bar{P}_T}{\partial t^2}}^{\,0} - \mu_0 \frac{\partial \bar{J}}{\partial t}$,

$$\nabla^2 \bar{E} = \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2} + \frac{\mu_0 \sigma}{1 - i\omega\tau}\frac{\partial \bar{E}}{\partial t}.$$

· For a simple harmonic oscillation of $\bar{E}$ ($E_0 e^{i(kz - \omega t)}$),

$$k^2 = \frac{\omega^2}{c^2} + \frac{i\omega\mu_0\sigma}{1 - i\omega\tau} \quad ; \; k : \text{complex}$$

i) For a very low frequency,

$$k^2 \sim i\omega\mu_0\sigma \longrightarrow k = \sqrt{i\omega\mu_0\sigma}$$

Because $i = \left(\frac{1+i}{\sqrt{2}}\right)^2$, $\sqrt{i} = \left(\frac{1+i}{\sqrt{2}}\right)$.

$$\therefore k = (1+i)\sqrt{\frac{\omega\sigma\mu_0}{2}} \rightarrow k_r = k_i = \sqrt{\frac{\omega\sigma\mu_0}{2}}.$$

ii) From $k = \frac{\omega}{c} n$ (6.29),

$$n = n_r + n_i = c\sqrt{\frac{\sigma\mu_0}{2\omega}} = \sqrt{\frac{\sigma}{2\omega\epsilon_0}}$$

(iii) From $\bar{E} = E_0 e^{i(kz - \omega t)} = E_0 e^{-k_i z} e^{i(k_r z - \omega t)}$,

The skin depth $\delta \equiv \frac{1}{k_i} = \sqrt{\frac{2}{\omega\sigma\mu_0}} = \sqrt{\frac{\lambda_0}{c\pi\sigma\mu_0}}$

$\delta^2 \propto \frac{1}{\sigma}$ (ex. Cu : $\delta \sim 0.1\,\mu m$) for microwave!

· From $k^2 = \frac{\omega^2}{c^2} + \frac{i\omega\mu_0\sigma}{1-i\omega\tau}$ & $k = \frac{\omega}{c}n$, ↙complex

$$n^2 = 1 + \left(\frac{c}{\omega}\right)^2 \frac{i\omega\mu_0\tau}{1-i\omega\tau}\left(\frac{Ne^2}{m}\right) \quad ; \; c^2 = \frac{1}{\mu_0\epsilon_0} \; ; \omega_p^2 = \frac{Ne^2}{m\epsilon_0}$$

$$= 1 + \frac{i\tau}{\omega(1-i\omega\tau)}\omega_p^2$$

From $\sigma = \frac{Ne^2}{m}\tau$,

$$= 1 + \frac{\omega_p^2}{-i\tau^{-1}\omega - \omega^2} = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\tau^{-1}}$$

· The plazma frequeny ω_p in a metal is

$$\omega_p = \sqrt{\frac{Ne^2}{m\epsilon_0}} = \sqrt{\frac{\sigma}{\tau\epsilon_0}} = \sqrt{\frac{\mu_0\sigma c^2}{\tau}}$$

· From

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\tau^{-1}} = 1 - \omega_p^2 \frac{1}{\omega^2 + i\omega\tau^{-1}} \frac{\omega^2 - i\omega\tau^{-1}}{\omega^2 - i\omega\tau^{-1}}$$

$$= 1 - \omega_p^2 \frac{\omega^2 - i\omega\tau}{\omega^2(\omega^2 + \tau^{-2})}$$

$$= 1 - \frac{\omega_p^2}{\omega^2 + \tau^{-2}} + i\frac{\omega_p^2\tau^{-1}}{\omega(\omega^2 + \tau^{-2})}$$

$$= n_r^2 - n_i^2 + i(2n_r n_i)$$

$$\therefore \; n_r^2 - n_i^2 = 1 - \frac{\omega_p^2}{\omega^2 + \tau^{-2}}$$

$$2n_r n_i = \left(\frac{1}{\omega\tau}\right)\frac{\omega_p^2}{\omega^2 + \tau^{-2}}$$

IR

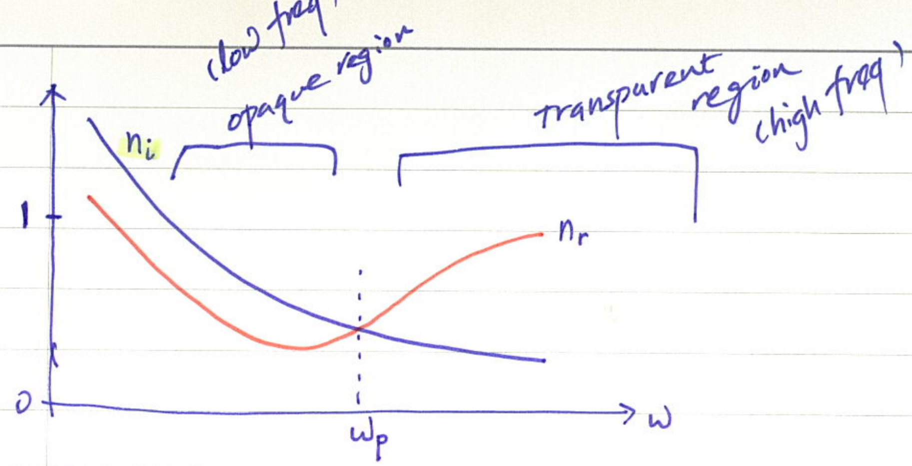
· Typical relaxatime time τ in metal : $\sim 10^{-13}$ s

· , plazma frequency $\frac{\omega_p}{2\pi}$ in metal : $\sim 10^{15}$ s^{-1}

VIS

Because τ^{-1} is damping γ,

$$\frac{\gamma}{\omega_p} \ll 1 .$$



- For poor conductors and semiconductors, oscillator strength $\sum_j f_j = 1$

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\tau^{-1}} + \omega_p^2 \sum_j \left(\frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j \omega} \right)$$

- From $\bar{J} = -Ne\bar{v}$ & $\bar{P} = -Ne\bar{r}$

$$= \frac{d\bar{P}}{dt} = -i\omega\bar{P}$$

6.10 Faraday rotation in solids

· In 1845, Faraday observed that

a plane of polarization of light is rotated by θ under $\underset{\text{static}}{\wedge}$ magnetic fields: $\theta = \underset{\text{Verdet const.}}{V} B l$

· $\bar{F} = -e\bar{E} - e\bar{v}\times\bar{B} = m\ddot{\bar{r}} + K\bar{r}$; $\bar{v} = \frac{d\bar{r}}{dt}$

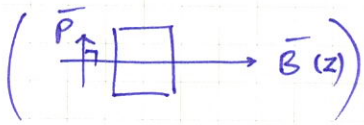
using $E = E_0 e^{-i\omega t}$,

$\rightarrow \quad -m\omega^2\bar{r} + K\bar{r} = -e\bar{E} + i\omega e\bar{r}\times\bar{B}$

Because polarization $\bar{P} = -Ne\bar{r}$,

$\rightarrow \quad (-m\omega^2 + K)\bar{P} = Ne^2\bar{E} + i\omega e\bar{P}\times\bar{B}$

For $\bar{r}\times\bar{B} = rB$,



$$\bar{P} = \left(\frac{Ne^2}{-m\omega^2 + K - i\omega eB}\right)\bar{E} \equiv \epsilon_0\chi\bar{E}$$

$$= \underbrace{\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{-\omega^2 + \frac{K}{m} - i\omega\left(\frac{eB}{m}\right)}\right)}_{\chi}\epsilon_0\bar{E}$$

· Double refraction (Voigt effect)

$\hat{z}$: No effect $\rightarrow \frac{Ne^2}{m\epsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2}\right)$; $\omega_0 = \sqrt{\frac{K}{m}}$ (resonance freq.)

$\hat{x}$ & $\hat{y}$: $\frac{Ne^2}{m\epsilon_0}\left(\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \omega^2\omega_c^2}\right)$; $\omega_c = \frac{eB}{m}$

imaginary (xy) : $\frac{Ne^2}{m\epsilon_0}\left(\frac{\omega\omega_c}{(\omega_0^2 - \omega^2)^2 + \omega^2\omega_c^2}\right)$

$$\frac{1}{\omega_0^2 - \omega^2 - i\omega\omega_c} = \frac{(\omega_0^2 - \omega^2) + i\omega\omega_c}{[(\omega_0^2 - \omega^2) - i\omega\omega_c][(\omega_0^2 - \omega^2) + i\omega\omega_c]}$$

6.11 Electro-optic effects

A. Kerr

- By strong electric field → double refraction occurs!

$$n_{\parallel} - n_{\perp} = K E^2 \lambda_0 \qquad : E \text{ is dc.}$$

K: Kerr const

- AC kerr effect : $\chi_{NL} \propto \chi^{(3)} E^2$: $\bar{P} = \chi^{(3)} E^3$

B. The Pockels effect

- n is altered by electric field (dc).
  → used for shutters or modulators.

6.12 Nonlinear optics

- $P = \epsilon_0 (\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \cdots)$
- $\bar{P} = \bar{P}^L + \bar{P}^{NL}$,

$$\bar{P}^L = \epsilon_0 \chi \bar{E} \quad ; \quad \bar{P}^{NL} = \epsilon_0 \chi^{(2)} \bar{E} \cdot \bar{E} + \epsilon_0 \chi^{(3)} \bar{E} \cdot \bar{E} \cdot \bar{E} + \cdots$$