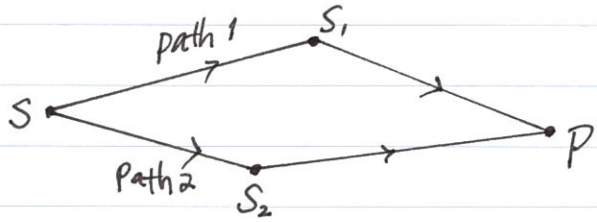
9-4. Partial coherence



· Let the source field be E.

Then

$$E_1(t) = \beta_1 E(t-T_1) = \beta_1 E_0 e^{-i\omega(t-T_1)} e^{i\phi(t-T_1)},$$

$$E_2(t) = \beta_2 E(t-T_2) = \beta_2 E_0 e^{-i\omega(t-T_2)} e^{i\phi(t-T_2)},$$

β_1 & β_2 : amplitude coefficients

T_1 & T_2 : traveling time along path 1 & 2

· $$I_p = \varepsilon_0 c \langle (E_{1p}+E_{2p})^2 \rangle = \varepsilon_0 c \{ \langle E_{1p}^2 \rangle + \langle E_{2p}^2 \rangle + 2 \langle E_{1p} E_{2p} \rangle \}$$

$$= I_{1p} + I_{2p} + \frac{\varepsilon_0 c}{2} \underbrace{\langle E_1 E_2 + E_1^* E_2^* + E_1 E_2^* + E_1^* E_2 \rangle}_{\text{interference}}$$

$\langle\ \rangle$ means time-average.

· $\langle E_1 E_2 \rangle = 0$; $\langle E_1^* E_2^* \rangle = 0$

$$\therefore I_p = I_{1p} + I_{2p} + \frac{\varepsilon_0 c}{2} \langle E_1 E_2^* + E_1^* E_2 \rangle$$

$$= I_{1p} + I_{2p} + \frac{\varepsilon_0 c}{2} 2\,\mathrm{Re}(\langle E_1 E_2^* \rangle)$$

$\downarrow$

$$\varepsilon_0 c\, \beta_1 \beta_2\, \mathrm{Re} \langle E(t-T_1) E^*(t-T_2) \rangle$$

Let $\tau = T_1 - T_2$ & make a time shift by T_1,

then $$I_p = I_{1p} + I_{2p} + \varepsilon_0 c \beta_1 \beta_2\, \mathrm{Re} \underbrace{\langle E(t) E^*(t+(T_1-T_2)) \rangle}_{\text{correlation fn}}$$

$$\Rightarrow \Gamma(\tau) \equiv \langle E(t) E^*(t+\tau) \rangle$$

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· Normalized correlation function $\gamma(\tau)$:

$$\gamma(\tau) \equiv \frac{\varepsilon_0 c \beta_1 \beta_2}{2} \frac{\Gamma(\tau)}{\sqrt{I_{1p} I_{2p}}} .$$

$$\therefore I_p = I_{1p} + I_{2p} + 2\sqrt{I_{1p} I_{2p}}\, \mathrm{Re}[\gamma(\tau)]$$

Here, $\gamma(\tau)$ is limited by the Source coherence time τ_0.
If $\tau > \tau_0$, no correlation → no interference,
where τ_0 is the coherence time of the source.

· For $I_p \propto \langle E E^* \rangle \rightarrow \frac{1}{2} E^2$, From (7-10).

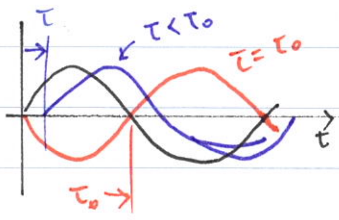
$$I_{1p} = \frac{\varepsilon_0 c}{2} (\beta_1 E_0)^2 \;;\; I_{2p} = \frac{\varepsilon_0 c}{2} (\beta_2 E_0)^2 ,$$

$$E(t) = E_0 e^{-i\omega t} e^{i\phi(t)}$$

$$\therefore \gamma(t) = \frac{\varepsilon_0 c}{2} \beta_1 \beta_2 \frac{\langle E(t) E^*(t+\tau) \rangle}{\sqrt{I_{1p} I_{2p}}}$$

$$= \frac{\varepsilon_0 c}{2} \beta_1 \beta_2 \frac{\langle E_0 e^{-i\omega t} e^{i\phi(t)} E_0 e^{i\omega(t+\tau)} e^{-i\phi(t+\tau)} \rangle}{\sqrt{\left(\frac{\varepsilon_0 c}{2}\right)^2 (\beta_1 \beta_2 E_0^2)^2}}$$

$$= e^{i\omega\tau} \underbrace{\langle e^{i(\varphi(t) - \varphi(t+\tau))} \rangle}_{=\ \frac{1}{T}\int_0^T e^{i[\varphi(t) - \phi(t+\tau)]} dt}$$



$$= e^{i\omega\tau} \left(1 - \frac{\tau}{\tau_0}\right)$$

$$\rightarrow |\gamma| = \left(1 - \frac{\tau}{\tau_0}\right) \qquad \therefore \gamma(t) = |\gamma| e^{i\omega\tau}$$

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$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

1. Complete incoherence: $\tau \to \tau_0$, $|\gamma| = 0$

   - $I_p = I_1 + I_2 = 2I_0$ (if $I_1 = I_2 = I_0$)
   - $V = \dfrac{2I_0 - 2I_0}{4I_0} = 0$

2. Complete coherence: $\tau = 0$, $|\gamma| = 1$

   - $I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\omega\tau$
   - $I_{max} = 4I_0$; $I_{min} = 0$
   - $V = \dfrac{4I_0}{4I_0} = 1$

3. Partial coherence: $0 < \tau < \tau_0$, $0 < |\gamma| < 1$

   - $I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\,Re(\gamma)$
     $= 2I_0[1 + Re(\gamma)]$
   - $I_{max} = 2I_0(1 + |\gamma|)$; $I_{min} = 2I_0(1 - |\gamma|)$
   - $V = \dfrac{4I_0|\gamma|}{4I_0} = |\gamma|$

Interpretation: The fringe visibility V is equal to the magnitude of correlation fn $|\gamma|$

→ measure of the degree of coherence.

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ex) $\lambda = 541\ nm$

$\Delta\lambda = 0.1\ nm\ (1\mathring{A})$

$\Delta L = 1.5\ mm$ (path difference btw'n two beams)

Q. What is V of the interference?

Sol) $V = 1 - \frac{\tau}{\tau_0} = 1 - \frac{\Delta L}{l_t}$; $l_t = \frac{\lambda^2}{\Delta\lambda}$ (9-18)

$= \frac{541^2}{0.1}(nm) = 2.93\ mm$

$\therefore V = 1 - \frac{1.5}{2.93} = 0.49$

For a double path difference,

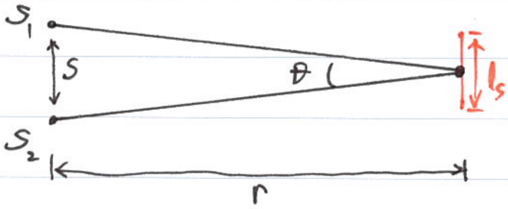
$\Delta L : 1.5 \rightarrow 3.0 > l_t$

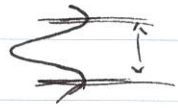
$\therefore V = 0$

9-5 Spatial coherence

- Temperal coherence → longitudinal coherence : $L = ct$
- Degree of coherence : by interference fringe contrast

- Spatial coherence → lateral coherence
  importace : phase btwn spatially distinct points
  → double slit (ex)





- Spatial coherence l_s :

$$l_s < \frac{\lambda}{\theta}$$

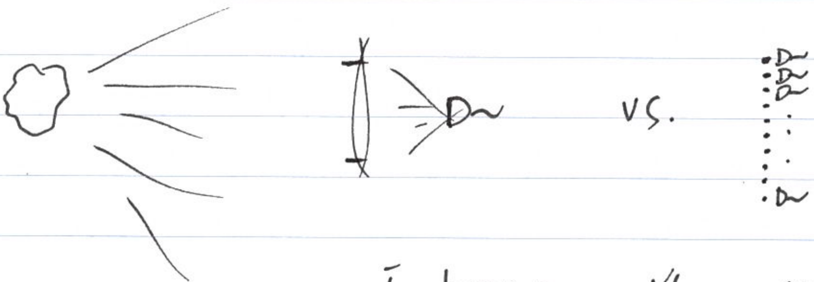
- In a double-slit exp,

$$y_m = \frac{m\lambda r}{s}$$

$$\Delta y = \frac{\lambda r}{s} = \frac{\lambda}{\theta}$$

$$\because \theta \sim \tan\theta = \frac{s}{r}$$

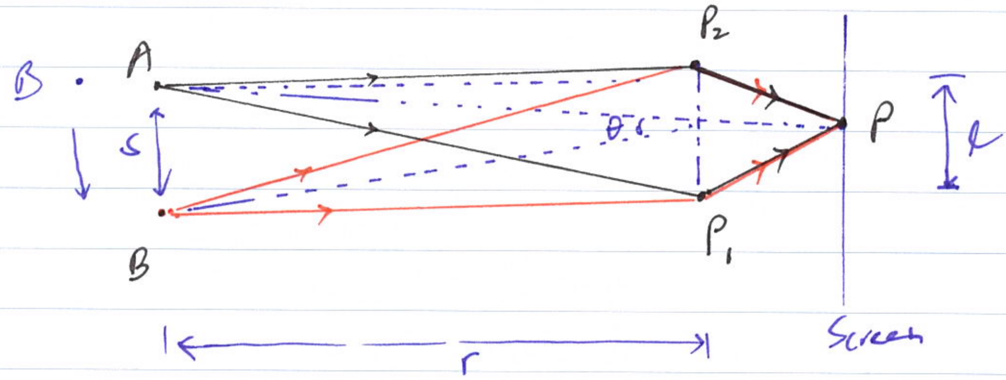
ex)



Incoherence vs coherence.

9-6. Spatial coherence width

· Two double slit exp.



· A & B : mutually incoherence light source
· P_1 & P_2 : double slit
· P : superposed point.

1. If max & min overlap each other → coherent!
2. If max of A overlaps min of B → incoherent!
3. Max in the interference occurs at the bisector of P_1 & P_2 for the case $S=0$ $(A=B)$.
   → $BP_2 - BP_1 = AP_2 - AP_1 = 0$
4. If source B is moved below A, $(S \neq 0)$ the system keeps coherence until

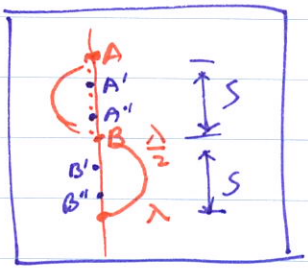
$$BP_2 - BP_1 = \Delta = \frac{\lambda}{2}$$

$$\rightarrow \Delta \sim l\theta \quad , \quad \theta \sim \frac{S}{r}$$

$$\therefore \Delta = \frac{\lambda}{2} = \frac{Sl}{r} \quad ; \quad S = \frac{r\lambda}{2l} \quad (\text{maximum } S)$$

5. If AB is replaced by an continuous array of point sources, the coherence (spatial) length l_s extends twice,

→ $l_s < \frac{r\lambda}{s} \sim \frac{\lambda}{\theta}$



6. If B moves away from A, then max occurs again.
→ periodic fringe pattern
→ will be discussed in CH. 11 (Diffraction)

ex) $r = 20$ cm (source to slit distance)
$l = 0.1$ mm (slit separation)
$\lambda = 546$ nm
Q. Determin the maximum width of the single slit

Sol) $s < \frac{r\lambda}{l_s} = \frac{(0.2)(546\times10^{-9})}{1\times10^{-4}} = 1.1$ mm