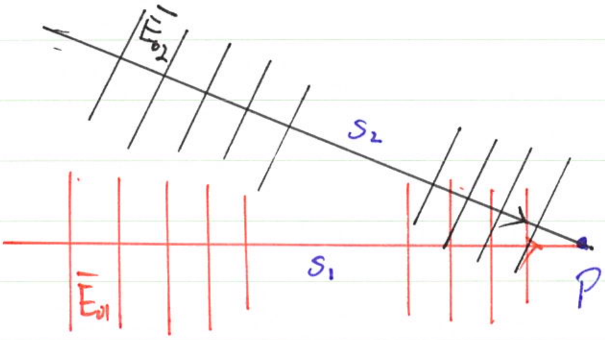
DATE 3/28/2016

CH. 7. Interference

7-1. Two-beam interference

- Two plane waves interference at P

$$\bar{E}_1 = \bar{E}_{01} \cos(ks_1 - \omega t + \phi_1)$$
$$\bar{E}_2 = \bar{E}_{02} \cos(ks_2 - \omega t + \phi_1)$$



$$\bar{E}_p = \bar{E}_1 + \bar{E}_2$$

- Irradiance

$$I = \varepsilon_0 c \langle \bar{E} \cdot \bar{E} \rangle \quad : \left(\frac{W}{m^2}\right) \text{ time-average}$$
$$= \varepsilon_0 c \langle \bar{E}_p^{\,2} \rangle = \varepsilon_0 c \langle (\bar{E}_1 + \bar{E}_2) \cdot (\bar{E}_1 + \bar{E}_2) \rangle$$
$$= \varepsilon_0 c \langle \bar{E}_1 \cdot \bar{E}_1 + \bar{E}_2 \cdot \bar{E}_2 + 2\bar{E}_1 \cdot \bar{E}_2 \rangle$$
$$= I_1 + I_2 + \underbrace{I_{12}}_{\text{interference.}}$$

(i) $\bar{E}_1 \perp \bar{E}_2$ → No interference effect!
(ii) $\bar{E}_1 \parallel \bar{E}_2$ → Maximum interference effect!

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$$I_{12} = 2\varepsilon_0 c \langle \bar{E}_1 \cdot \bar{E}_2 \rangle$$

$\rightarrow \bar{E}_1 \cdot \bar{E}_2 = \bar{E}_{01} \cdot \bar{E}_{02} \cos(ks_1 - \omega t + \phi_1)\cos(ks_2 - \omega t + \phi_2)$

Let $\alpha \equiv ks_1 + \phi_1$ & $\beta \equiv ks_2 + \phi_2$

$\cdot$ $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$\rightarrow 2\langle \bar{E}_1 \cdot \bar{E}_2 \rangle = \bar{E}_{01} \cdot \bar{E}_{02} \left[\underbrace{\langle \cos(\alpha+\beta-2\omega t) \rangle}_{0 \text{ for the time average}} + \langle \cos(\alpha-\beta) \rangle \right]$

$\therefore 2\langle \bar{E}_1 \cdot \bar{E}_2 \rangle = \bar{E}_{01} \cdot \bar{E}_{02} \langle \cos(\alpha-\beta) \rangle$

$= \bar{E}_{01} \cdot \bar{E}_{02} \langle \cos\delta \rangle,$

$\delta \equiv k(s_1 - s_2) + (\phi_1 - \phi_2)$

For random phases of ϕ_i (if purely monochromatic),

$\langle \cos\delta \rangle = \cos\delta.$

For real field,

$$I_{12} = \varepsilon_0 c \, \bar{E}_{01} \cdot \bar{E}_{02} \langle \cos\delta \rangle$$

Because $I_1 = \varepsilon_0 c E_{01}^2 \langle \cos^2(\alpha - \omega t) \rangle = \frac{1}{2}\varepsilon_0 c E_{01}^2,$

$I_2 = \varepsilon_0 c E_{02}^2 \langle \cos^2(\beta - \omega t) \rangle = \frac{1}{2}\varepsilon_0 c E_{02}^2,$

$$I_{12} = 2\sqrt{I_1 I_2} \langle \cos\delta \rangle.$$

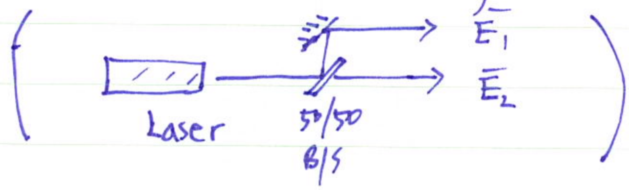
For random δ, or random sources,

$\langle \cos\delta \rangle = 0.$

$\rightarrow$ E_1 & E_2 are mutually <u>incoherent</u>!

& $I = I_1 + I_2.$

⟨Interference of mutually coherent beams⟩



· E_1 & E_2 are perfectly coherent!

· $I_{12} = 2\sqrt{I_1 I_2}\langle \cos\delta \rangle \neq 0. \rightarrow 2\sqrt{I_1 I_2}\cos\delta$

· δ depends on the path length difference!

· coherence time τ_0,

$$\tau_0 = \frac{1}{\Delta\nu},$$

$\Delta\nu$: bandwidth of the laser.

· coherence length l_c,

$$l_c = c\tau_0$$

$\therefore$ δ also depends on actual path length of the laser beam.

ex) $\Delta\nu = 1\,\text{MHz} \rightarrow \tau_0 = 1\,\mu s$

$\rightarrow l_c = 300\,m$

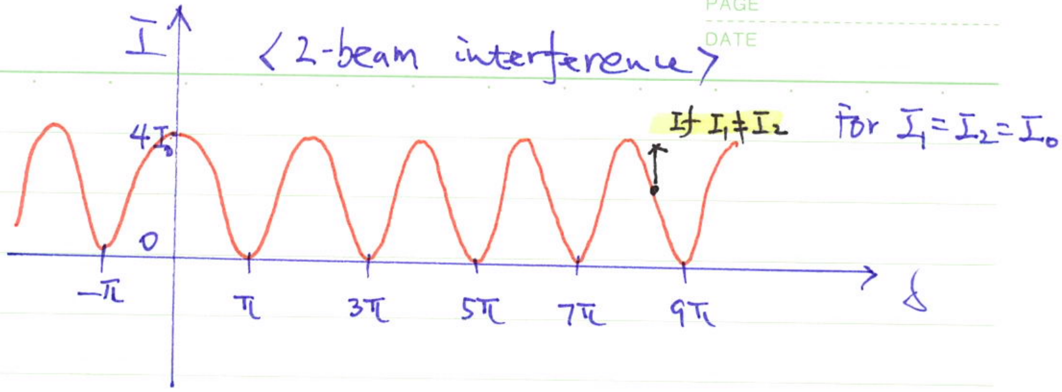
· Constructive interference : Max irradiation. $(\cos\delta = 1)$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \rightarrow 4I_0 \; (\text{if } I_1 = I_2 = I_0)$$

· Destructive interference : Min irradiation $(\cos\delta = -1)$

$$I = I_1 + I_2 - 2\sqrt{I_1 I_2} \rightarrow 0 \; (\text{if } I_1 = I_2 = I_0)$$

<2-beam interference>



- Visibility $= \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$ (fringe contrast)

- For the same amplitude,

$$I = I_0 + I_0 + 2\sqrt{I_0^2}\cos\delta = 2I_0(1+\cos\delta)$$

with $1 + \cos\delta = 2\cos^2\left(\frac{\delta}{2}\right)$,

$$I = 4I_0\cos^2\left(\frac{\delta}{2}\right)$$: Two-beam interference when $I_1 = I_2 = I_0$

- Energy conservation is violated at a specific point!

→ However,

$$I_{average} = 2I_0 \quad , \quad \langle\cos^2\alpha\rangle = \frac{1}{2}$$

∴ Interferance satisfies energy conservation law!